The Analysis of the Augmented ART1 Neural Network

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Abstract

The augmented ART1 neural network was introduced in a companion paper. The dynamics of the AART1-NN are described by a set of nonlinear differential equations that facilitate the real time implementation of the ART1 neural network. In this paper we show that under certain parameter constraints the AART1 model behaves in the same manner as the ART1 model.

1 Introduction

The augmented ART1 neural network (AART1-NN) was introduced in a companion paper ([1]). The AART1-NN is described by a set of nonlinear differential equations that are extensions to those developed by Carpenter and Grossberg in [2], where the ART1 neural network (ART1-NN) was introduced. The importance of the AART1-NN is that it facilitates the real time implementation of the ART1-NN. The term real time is used in its strictest sense. That is, in the AART1 model we have removed all algorithmic components from an implementation of the ART1 model. This feature of the AART1-NN offers the inherent advantage of flexible network implementation and it also allows the network to function in either the slow or the fast learning case. Algorithmic implementations of ART1 do not exhibit these capabilities. The dynamics of the AART1-NN are completely determined by a set of differential equations that comprise the entire model. The behavior of the differential equations that define the AART1-NN depends on a number of parameters. In this paper we show that under certain constraints on these parameters the operation of the AART1-NN is identical with the operation of the ART1-NN. Since the AART1 model is an extension of the ART1 model, this work proves the inherent capability of the ART1-NN to operate in a totally unsupervised manner.

2 Preliminaries

In the following sections we introduce useful notation, and we briefly summarize the AART1-NN equations.

2.1 Notation

The architecture of the AART1-NN and its major components is presented in Figure 1. For notation purposes we need to know that the AART1 model consists of two fields of nodes denoted F1 and F2. We denote nodes in the F1 field by v_i and nodes in the first layer of the F2 field by v_j. The index of the nodes in the F1 field ranges from 1 to M, while the index of the nodes in the first layer of the F2 field ranges from $M+1$ to N. For every node v_j in the first layer of the F2 field there is a corresponding inhibitory node $\hat{v}_j$ that resides in the second layer of the F2 field. The index of the inhibitory nodes in the F2 field ranges from $M+1$ to N. We denote the activity of node v_i in the F1 field by x_i, the activity of a node v_j in the F2 field by x_j and the activity of an inhibitory node $\hat{v}_j$ in the F2 field by $\hat{x}_j$. We also denote by z_{ij} the value of the bottom up LTM trace associated with an arc connecting node v_i in the F1 field with node v_j in the F2 field. We finally denote by z_{ji} the value of the top down LTM trace associated with an arc connecting node v_j in the F2 field with node v_i in the F1 field. The reset mechanism in the AART1 model is initiated by the reset node v_r whose activity is denoted by x_r. Let us also introduce the following vectors.

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$$I = (I_1, \ldots, I_M)$$

$$X = (x_1, \ldots, x_M)$$

$$S = (f_1(x_1), \ldots, f_1(x_M))$$

$$T = (T_{M+1}, \ldots, T_N) = (D_2 \sum_i z_{i,M+1} f_1(x_i), \ldots, D_2 \sum_i z_{iN} f_1(x_i))$$

$$Y = (x_{M+1}, \ldots, x_N)$$

$$U = (f_2(x_{M+1}), \ldots, f_2(x_N))$$

$$V = (V_1, \ldots, V_M) = (D_1 \sum_j z_{j1} f_2(x_j), \ldots, D_1 \sum_j z_{jM} f_2(x_j))$$

I corresponds to the input pattern at the F1 field. Its components are referred to as the *bottom up inputs* at the F1 field. X is the STM activity across the F1 field. S is the *output pattern* across the F1 field. T is the *bottom up input* from the F1 field that affects the first layer of the F2 field. Y is the *STM activity* across the first layer of the F2 field. U is the *output pattern* across the first layer of the F2 field. V is the *top down input* from the first layer of the F2 field that affects the F1 field. Its components are referred to as the *top down* inputs at the F1 field.

A node with activity below or above the quenching threshold is denoted as *subliminally* or *supraliminally* active, respectively. The quenching threshold is a small positive constant. We say that a node in the network is *activated* if its activity increases from a level below the quenching threshold to a level above the quenching threshold. We also say that a node in the network is *deactivated* if its activity drops from a level above the quenching threshold to a level below the quenching threshold.

A *contrast enhancement* or *competition* cycle is the process that the first layer of the F2 field goes through to choose one node that will accurately represent the input pattern I presented across the F1 field. In this paper, the only case considered is the one where only one node in the first layer of the F2 field is chosen to represent the input pattern I. If after the activation of a node in the first layer of the F2 field no reset occurs we say that this node *codes* the input pattern I.

2.2 The AART1 Neural Network Equations

In the following, we present the differential equations that define the AART1 model. These equations are also presented in more detail in [1] where the AART1 model is introduced. The activity of a node v_i in the F1 field satisfies the following differential equation:

$$\epsilon_1 \frac{d}{dt} x_i = -x_i + (1 - A_1 x_i) J_i^+ - (B_1 + C_1 x_i) J_i^- \tag{1}$$

where,

$$J_i^+ = I_i + D_1 \sum_j f_2(x_j) z_{ji} \tag{2}$$

$$J_i^- = \sum_j f_2(x_j). \tag{3}$$

$$f_2(x_j) = \begin{cases} 1, & \text{if } x_j > \delta_2; \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

The activity of the reset node v_r satisfies the following differential equation:

$$\epsilon_r \frac{d}{dt} x_r = -A_r x_r + \mathrm{U}[\rho \sum_{i=1}^{M} I_i - \sum_{i=1}^{M} f_1(x_i)] \tag{5}$$

where U is the unit step function ($U(x) = 0$ for $x \leq 0$ and $U(x) = 1$ for $x > 0$). The activity of an inhibitory node $\hat{v}_j$ in the F2 field satisfies the following differential equation:

$$\epsilon_2 \frac{d}{dt}\hat{x}_j = -[1 - g(I)]\hat{x}_j + g(I) f_r(x_r) f_2(x_j) \tag{6}$$

where

$$g(I) = \begin{cases} 1, & \text{if } \sum_{i=1}^{M} I_i \neq 0; \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

$$f_r(x_r) = \begin{cases} 1, & \text{if } x_r > \delta_r; \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The activity of a node v_j in the F2 field satisfies the following differential equation:

$$\epsilon_2 \frac{d}{dt} x_j = -x_j + (1 - A_2 x_j) J_j^+ - (B_2 + C_2 x_j) J_j^- \tag{9}$$

where

$$J_j^+ = f_2(x_j) g(I) + T_j \tag{10}$$

$$T_j = D_2 \sum_i f_1(x_i) z_{ij} \tag{11}$$

$$J_j^- = \sum_{k \neq j} f_2(x_k) + \hat{f}_2(\hat{x}_j). \tag{12}$$

$$f_1(x_i) = \begin{cases} 1, & \text{if } x_i > \delta_1; \\ 0, & \text{otherwise,} \end{cases} \tag{13}$$

$$\hat{f}_2(\hat{x}_j) = \begin{cases} 1, & \text{if } \hat{x}_j > \hat{\delta}_2; \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

The value of the bottom-up LTM trace, z_{ij}, is determined by the following differential equation:

$$\epsilon_z \frac{d}{dt} z_{ij} = -K[(L-1) f_1(x_i) + \sum_k f_1(x_k)] f_2(x_j) z_{ij} + KL\, f_1(x_i) f_2(x_j) \tag{15}$$

The value of the top down LTM trace, z_{ji}, is determined by the following differential equation:

$$\epsilon_z \frac{d}{dt} z_{ji} = -f_2(x_j) z_{ji} + f_1(x_i) f_2(x_j) \tag{16}$$

3 Operation of the AART1 Neural Network

The behavior of the differential equations that define the AART1 model (see Section 2) depends on a number of parameters that are listed in Table 1. In this section we show that there exists a set of parameter values for which the AART1-NN operates like the ART1-NN. The approach to accomplish this task is as follows: We state three postulates (1,2,3). The satisfaction of these postulates guarantees the successful operation of the AART1-NN (i.e., an operation identical with the ART1-NN operation). The postulates are valid under certain parameter constraints, and as a result, these parameter constraints guarantee the successful operation of the AART1-NN.

It is worth noting that during the operation of the AART1-NN we make the assumption that the zero pattern is briefly presented at the F1 field in between any two successive nonzero input pattern

	Parameters
F1 field	$A_1\ B_1\ C_1\ D_1\ \epsilon_1\ \delta_1$
F2 field	$A_2\ B_2\ C_2\ D_2\ \epsilon_2\ \delta_2\ \hat{\delta}_2$
Reset System	$A_r\ \epsilon_r\ \delta_r\ \rho$
LTM traces	$K\ L\ \epsilon_z$

Table 1: List of parameter values in the AART1-NN model

presentations to the network. This allows all the node activities in the AART1-NN to return to their resting value of zero prior to any nonzero pattern presentation at the F1 field of the network. Furthermore, it disengages the enduring inhibition of the nodes in the first layer of the F2 field that have been reset during a nonzero pattern presentation, and it allows the F2 field of the AART1-NN to code the next nonzero input pattern without bias.

Let us now consider the presentation of a nonzero input pattern I at the F1 field of the AART1-NN. Let us also denote by, O_{M+1}, O_{M+2}, ..., O_{N-1}, O_N, the bottom-up inputs that affect the nodes v_{M+1}, v_{M+2}, ..., v_{N-1}, respectively, when the output activity at the F1 field is equal to I. The following postulates guarantee the successful operation of the AART1-NN during the presentation of pattern I at the F1 field of the AART1-NN.

Postulate 1:
The reset node is activated only if there is a mismatch between bottom up and top-down inputs. The reset node is subliminally active for the time interval during which an active node in the first layer of the F2 field is tested for its appropriateness to represent the input pattern I (i.e., from the time that this node becomes active until the output activity at the F1 layer changes from I to $I \cap V$, where V stands for the top-down input emanating from this node).

Postulate 2:
After a reset event the output activity at the F1 field returns to I long before another node (i.e., different than the ones that have already been reset) in the first layer of the F2 field becomes active. This postulate guarantees that competition in the first layer of the F2 field occurs, primarily, during the time intervals at which the output activity at the F1 layer is equal to I.

Postulate 3:
During the presentation of the nonzero pattern I at the F1 field of the AART1-NN the nodes in the first layer of the F2 field are going to be activated, if necessary, with the following order v_{M+1}, v_{M+2}, ..., v_{N-1}. Some nodes may not even get the chance to be activated in the case that a previously activated node is found to be the appropriate one to represent the input pattern I.

Postulates 1,2,3 are valid under constraints CON1-CON24 that are listed below. In constraints CON1-CON24, $O_{\max}$ corresponds to an upper bound on the O_j's for any input pattern presented at the F1 field of the AART1-NN, $O_{\min}$ is a lower bound on the O_j's for any input pattern presented at the F1 field of the AART1-NN, $z_{ij}(0)$ and $z_{ji}(0)$ are the bottom up and top down traces, respectively, prior to the presentation of the first input pattern at the F1 field of the AART1-NN.

CON1: The O_j's $j = M+1, \ldots, N$ are distinct.
CON2: $\epsilon_1, \epsilon_2, \epsilon_r << \epsilon_z$
CON3: $A_1 > 0,\ C_1 > 0,\ A_2 > 0,\ C_2 > 0$
CON4: $B_1 > 0,\ D_1 > 0,\ B_2 > 0,\ D_2 > 0$
CON5: $0 < \rho \leq 1$
CON6: $L > 1,\ \ 0 < z_{ij}(0) < \frac{L}{L-1+M}$
CON7: $\frac{B_1 - 1 + \delta_1(1 + A_1 + C_1)}{D_1(1 - \delta_1 A_1)} < z_{ji}(0) \leq 1$

CON8: $\max\{1, D_1\} - \delta_1(1 + A_1 + C_1) < B_1 < 1 + D_1 - \delta_1(1 + A_1 + D_1 A_1 + C_1)$

CON9: $\delta_r > \frac{1-[1-\delta_1(1+A_1)]^{\frac{A_r \epsilon_1}{(1+A_1)\epsilon_r}}}{A_r}$

CON10: $B_2 > O_{\max}$

CON11: $B_2 > 1 + O_{\max}$

CON12: $\delta_1 = 0.01,\ A_1 = 1.0,\ \epsilon_2 = 0.01$

CON13: $D_1 = 1,\ B_1 = 0.5,\ C_1 = 100$

CON14: $A_2 O_{\max} < A_1,\ \epsilon_1 = 0.1\epsilon_2$

CON15: $B_2 = \alpha O_{\max}^2,\ \ C_2 = \alpha O_{\max}$ with $\alpha > 0$

CON16: $\delta_2 = 0.01,\ A_2 = 0.3$

CON17: $B_2 = \beta O_u^2,\ C_2 = \beta O_u$ where $\beta \geq \frac{5(1+A_1+D_1 A_1+C_1)\epsilon_2}{\epsilon_1}$, $O_u = \max(O_{\max}, 1)$

CON18: $1 + A_2 O_{\max} << C_2$

CON19: $\delta_2 << B_2 C_2^{-1}$

CON20: $A_r = 2,\quad \delta_r = 0.02,\quad \epsilon_r = \epsilon_1$

CON21: $\frac{0.99792}{O_{j_2}} - \frac{1}{O_{j_1}} > (1 - 0.99792) \cdot A_2$ for $j_1 = M + 2, \ldots, N - 1$ and $j_2 > j_1$

CON22: $\frac{1}{1+A_1} > \delta_1$

CON23: $\frac{O_{\min}}{1+A_2 O_{\min}} > \delta_2$

CON24: $\frac{1}{A_r} > \delta_r$

CON1 is required for the successful operation of the ART1-NN (see [2]), and as a result, for the successful operation of the AART1-NN. CON2 implies that the LTM traces in the AART1-NN change very slowly compared to the activities of the nodes in the network. CON3 guarantees that the activity x_i will be constrained in the interval $[-B_1 C_1^{-1}, A_1^{-1}]$ and the activity x_j will be constrained in the interval $[-B_2 C_2^{-1}, A_2^{-1}]$. CON4 and the fact that the z_{ij}'s, z_{ji}'s are nonegative makes sure that J_i^+, J_j^+ are indeed excitatory signals and that J_i^-, J_j^- are indeed inhibitory signals. CON5 is required for the successful operation of the reset mechanism described in detail in [2]. CON6 is required for the satisfaction of the direct access inequality (see [2]), while CON7 and CON8 are necessary for the validity of the $\frac{2}{3}$ rule (see [2]). CON9-CON21 are introduced to guarantee the validity of Postulates 1,2,3. CON22-CON24 impose a lower bound on the forcing terms for the differential equations that describe node activities in the AART1-NN. If these lower bounds are not imposed the activities of the nodes in the AART1-NN would never exceed their quenching thresholds. The proof that, under constraints CON1-CON24, Postulates 1,2,3 are valid is carried out in every detail in [3]; it is ommitted here due to lack of space.

It is worth noting that under constraints CON1-CON24 when one node becomes supraliminally active in the first layer of the F2 field the activities of the other nodes are rapidly driven to their extreme negative values independently of the bottom up inputs that they receive from the F1 field. It is also worth noting that the activity of a node in the F1 field that receives bottom up and strong top down input reaches a limiting value that is above the quenching threshold but very close to the quenching threshold. Finally, the activity of a node in the F1 field that receives bottom up input and weak top down input reaches a limiting value that is below the quenching threshold but very close to the quenching threshold.

4 Conclusions

In this work we provided a summary of results from a detailed analysis of the differential equations that define the AART1-NN model. The analysis of the differential equations that define the AART1-NN model (for more details see [3]) demonstrated how the AART1-NN parameters can be chosen (see Section 3) in order to guarantee that the AART1 model behaves in the same manner as the ART1 model. The AART1-NN is useful because it facilitates the real time implementation of the ART1-NN.

It should be emphasized that the validity of Postulates 1,2,3, under constraints CON1-CON24, is independent of whether the AART1-NN operates in a slow learning environment or a fast learning environment. As a result, under constraints CON1-CON24, the AART1-NN operates like the ART1-NN in either the fast or the slow learning cases. In a fast learning environment the input pattern is presented long enough for the bottom up and top down traces to reach their limiting values. A slow learning environment corresponds to the case where the input pattern is presented long enough for the the network to choose the right node in the first layer of the F2 field, that will code the input pattern, but not necessarily long enough for the bottom up and top down traces to reach their limiting values.

The set of constraints (CON1-CON24), derived in Section 3, that guarantee the successful operation of the AART1-NN is not the only set for which the AART1-NN behaves in a desirable way (i.e., like the ART1-NN). It is though a set of constraints for which we could prove analytically that the AART1-NN exhibits the desirable behavior. If another set of constraints were to be constructed they had to be such that Postulates 1,2,3 are true. Hence, the formulation of Postulates 1,2,3, in Section 3, provides the right framework in an effort to find constraints on the AART1-NN parameter values that will guarantee its successful operation.

Acknowledgement: This research was supported by a grant from the Florida High Technology and Industry Council.

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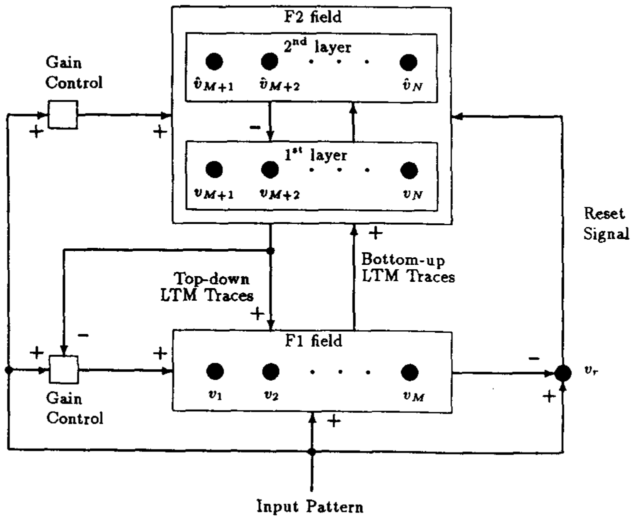

Figure 1: The architecture of the AART1-NN model.